Probability Function

A Poisson process is a situation in which a phenomenon occurs at a constant average rate. Each occurrence is independent of all other occurrences; in a Poisson process, an event does not become more likely to occur just because it's been a long time since its last occurrence. The location of potholes on a highway or the emission over time of particles from a radioactive substance may be Poisson processes.

The probability density function:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & x < 0 \end{cases}$$

describes the relative likelihood of an occurrence at time or position x, where λ describes the average rate of occurrence.

The probability $P(a < x < b)$ of an event occurring in the interval between a and b is given by:

$$\int_a^b f(x)\,dx.$$

Compute this integral:

a) for the case in which a and b are both positive (assume $a < b$),

b) for the case in which $a \leq 0$ and $b > 0$,

c) for the case in which $a \leq 0$ and $b \leq 0$.

Probability Function

A Poisson process is a situation in which a phenomenon occurs at a constant average rate. Each occurrence is independent of all other occurrences; in a Poisson process, an event does not become more likely to occur just because it's been a long time since its last occurrence. The location of potholes on a highway or the emission over time of particles from a radioactive substance may be Poisson processes.

The probability density function:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & x < 0 \end{cases}$$

describes the relative likelihood of an occurrence at time or position x, where λ describes the average rate of occurrence.

The probability $P(a < x < b)$ of an event occurring in the interval between a and b is given by:

$$\int_a^b f(x)\, dx.$$

9/8/25

Compute this integral:

a) for the case in which a and b are both positive (assume $a < b$),

b) for the case in which $a \leq 0$ and $b > 0$,

c) for the case in which $a \leq 0$ and $b \leq 0$.

a) $\int_a^b f(x)\,dx$

$= \int_a^b \lambda e^{-\lambda x}\,dx$

$= \lambda \left(\frac{e^{-\lambda x}}{-\lambda} \right) \Big|_a^b$

$= -1 \left(e^{-\lambda b} - e^{-\lambda a} \right)$

$= e^{-a\lambda} - e^{-b\lambda}$

b) $\int_a^b f(x)\,dx$

$= \int_a^0 0\,dx + \int_0^{a=0} \lambda e^{-\lambda x}\,dx$

$+ \int_a^b \lambda e^{-\lambda x}\,dx$

$= 0 + 0 + \frac{\lambda e^{-\lambda x}}{-\lambda} \Big|_0^b$

$= -1 \left(e^{-\lambda b} - e^{0} \right)$

$= 1 - e^{-b\lambda}$

c) $\int_a^b f(x)\,dx$

$= \int_0^{b=0} \lambda e^{-\lambda x}\,dx + \int_a^0 0\,dx$

$+ \int_a^b 0\,dx$

$= 0 + 0 + 0$

$= 0$